exercice 6 :

1) b) $u_k \geq \frac{15}{4} \times 0,5^k$

$$\frac{1}{5} u_k \geq \frac{3}{4} \times 0,5^k$$

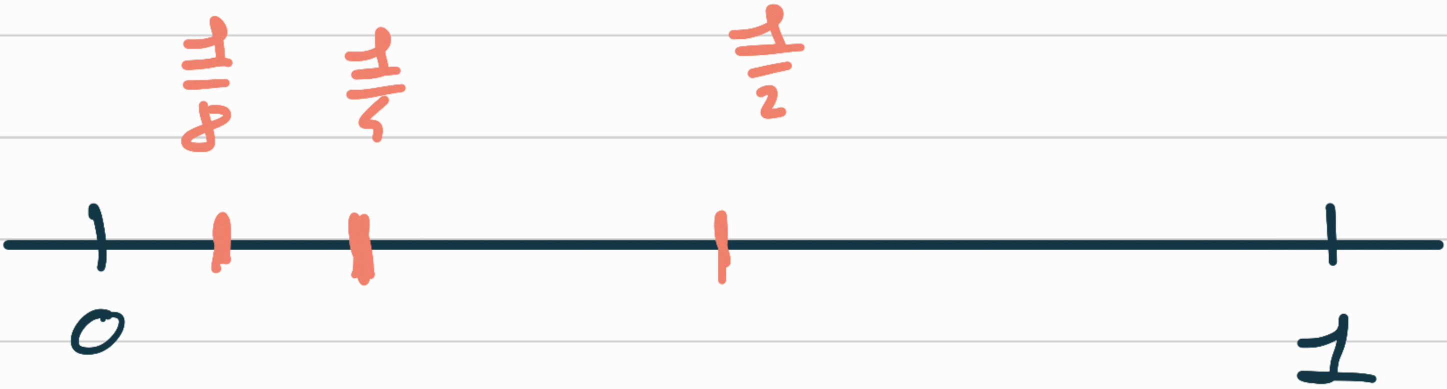
$$\frac{1}{5} u_k + 3 \times 0,5^k \geq \frac{3}{4} \times 0,5^k + 3 \times 0,5^k$$

$$u_{k+1} \geq \left(\frac{3}{4} + 3\right) \times 0,5^k$$

$$u_{k+1} \geq \frac{15}{4} \times 0,5^k$$

k+1 ?

or $0,5^k \geq 0,5^{k+1}$



d'où $\quad u_{k+1} \geqslant \frac{15}{4} \times 0,5^{k+1}$

$1 \geqslant 0,5$

$$u_{k+1} \times 1 \geqslant \frac{15}{4} \times \underbrace{0,5^{k} \times 0,5}_{0,5^{k+1}}$$

2) $u_{n+1} - u_n$

$= \frac{1}{5}u_n + 3 \times 0{,}5^n - u_n$

$\frac{1}{5} - 1 = -\frac{4}{5}$

$= \boxed{-\frac{4}{5}u_n + 3 \times 0{,}5^n}$

or $u_n \geqslant \frac{15}{4} \times 0{,}5^n$

$\times -\frac{4}{5}$

$-\frac{4}{5}u_n \leqslant -\frac{4}{5} \times \frac{15}{4} \times 0{,}5^n$

$-\frac{4}{5}u_n \leqslant -3 \times 0{,}5^n$

$\boxed{-\frac{4}{5}u_n + 3 \times 0{,}5^n} \leqslant \underbrace{-3 \times 0{,}5^n + 3 \times 0{,}5^n}_{=0}$

$u_{n+1} - u_n \leqslant 0$

exercice 7 :

$$\begin{cases} u_0 = 4 \\ u_{n+1} = \frac{1}{2}\left(u_n + \frac{5}{u_n}\right) \end{cases}$$

1) $u_k > 0$ (H.R)

de plus $\frac{5}{u_k} > 0$

donc $u_k + \frac{5}{u_k} > 0 + 0 = 0$

$$\frac{1}{2}\left(u_k + \frac{5}{u_k}\right) > \frac{1}{2} \times 0 = 0$$

$$u_{k+1} > 0$$

2) $u_k - \sqrt{5} \geq 0$

$u_k \geq \sqrt{5}$

$$0 \leq \frac{5}{u_k} \leq \frac{5}{\sqrt{5}}$$

$$u_{n+1} - \sqrt{5} = \frac{1}{2}\left(u_n + \frac{5}{u_n}\right) - \sqrt{5}$$

$$= \frac{1}{2}\left(\frac{u_n^2 + 5}{u_n}\right) - \sqrt{5}$$

$$= \frac{u_n^2 + 5}{2u_n} - \sqrt{5}$$

$$= \frac{u_n^2 + 5 - \sqrt{5} \times 2u_n}{2u_n} \quad \geq 0$$

$$= \frac{\overset{a^2}{u_n^2} - \overset{2 \times a \times b}{2 \times u_n \times \sqrt{5}} + \overset{b^2}{5}}{2u_n}$$

$$= \frac{(u_n - \sqrt{5})^2}{2u_n} \quad \begin{matrix} \rightarrow \geq 0 \text{ par H.R} \\ \geq 0 \quad \text{qu. 1} \end{matrix}$$

$$\geq 0$$

exercice 8 :

$$0 < v_n < 3$$

$$-3 < -v_n < 0$$

$$0 < 3 < 6 - v_n < 6$$

Inégalité de Bernoulli :

$$\forall n \in \mathbb{N}, \quad (1+a)^n \geq 1 + na$$

Hérédité : On suppose que

$$(1+a)^k \geq 1 + ka$$

On veut montrer que $(1+a)^{k+1} \geq 1+(k+1)a$

$$(1+a)^{k+1} = \overbrace{(1+a)^k}^{H.R} \times (1+a)$$

$$\geq (1+ka)(1+a)$$

$$\geq \underbrace{1 + a + ka}_{\parallel} + ka^2$$

or $1 + (k+1)a = \boxed{1 + ka + a}$

$$\geq 1 + (k+1)a + ka^2 \qquad \geq 0$$

$\geq 1 + (k+1)a$

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exercice 38 :

pair $\Rightarrow u_k = 2 \times q \ , q \in \mathbb{Z}$

impair $\Rightarrow u_k = 2q + 1, \ q \in \mathbb{Z}$

$\begin{cases} u_k \text{ multiple de } 3 \\ 3 \text{ divise } u_k \end{cases} \Rightarrow u_k = 3q, q \in \mathbb{Z}$

a) Supposons que u_k pair.

donc il existe $q \in \mathbb{Z}$, tel que :

$$u_k = 2q$$

or $u_{k+1} = u_k + 6k$

$$= 2q + 2 \times 3k$$

$$= 2\underbrace{(q + 3k)}_{\in \mathbb{Z}}$$

donc u_{k+1} est pair.

b) Supposons u_k multiple de 3.

Il existe $q \in \mathbb{Z}$, $u_k = 3q$

$$u_{k+1} = u_k + 6k$$
$$= 3q + 3 \times 2k$$
$$= 3\underbrace{(q + 2k)}_{\in \mathbb{Z}}$$

donc u_{k+1} multiple de 3.

$$u_n \geqslant n$$

$$3u_n \geqslant 3n$$

$$3u_n - 2n \geqslant 3n - 2n$$

$$3u_n - 2n \geqslant n$$

$$3u_n - 2n + 3 \geqslant n + 3$$

$$u_{n+1} \geqslant n + 3 \geqslant n$$

$$u_{n+1} = 3u_n - 2n + 3$$

$\downarrow$ H.R

$$= 3(3^n + n - 1) - 2n + 3$$

$$= 3^{n+1} + 3n - 3 - 2n + 3$$

$$= 3^{n+1} + n \quad ✓$$