Forme Canonique

$$f(x) = ax^2 + bx + c$$

1, Forme canonique :

$$f(x) = a(x - \alpha)^2 + \beta$$

① Méthode astucieuse

$f(x) = 4x^2 - 12x + 18$

$a = 4 \quad b = -12$

$c = 18$

↳ essayer de reconnaître le début d'une identité $(a+b)^2$ / $(a-b)^2$

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

↳ $2 \times 2x \times ?$

$= (2x - 3)^2 + 18$

↳ $(2x)^2 = 4x^2$

↳ $(2x-3)^2 = (2x)^2 - 2 \times 2x \times 3 + 3^2$

$= 4x^2 - 12x + 9$ (9 : en trop)

$$= (2x - 3)^2 - 9 + 18$$

↳ toujours enlever le 3^2

$$= (2x - 3)^2 + 9$$

↳ factoriser le chiffre devant le x

$$= \left(2\left(x - \frac{3}{2}\right)\right)^2 + 9$$

$$= 2^2\left(x - \frac{3}{2}\right)^2 + 9$$

$$= 4\left(x - \frac{3}{2}\right)^2 + 9$$

② Méthode Simple

$$f(x) = ax^2 + bx + c$$

↳ $f(x) = a(x-\alpha)^2 + \beta$

$$\begin{cases} \alpha = -\dfrac{b}{2a} \\ \beta = f(\alpha). \end{cases}$$

$$f(x) = 4x^2 - 12x + 18$$

↳ $a = 4$, $b = -12$, $c = 18$

$$\alpha = -\frac{b}{2a} = -\frac{(-12)}{2\times 4} = \frac{12}{8}$$

$$= \frac{3}{2}$$

$$\beta = f(\alpha) = f\left(\frac{3}{2}\right)$$

$$= 4\times\frac{3}{2}^{2} - 12\times\frac{3}{2} + 18$$

$$= 9$$

$$f(x) = 4\left(x-\frac{3}{2}\right)^{2} + 9$$

exercice :

- $f(x) = 2x^2 - 20x + 10$

↳ $a = 2 \quad b = -20 \quad c = 10$

$$\alpha = -\frac{b}{2a} = \frac{-(-20)}{2 \times 2} = \frac{20}{4} = 5$$

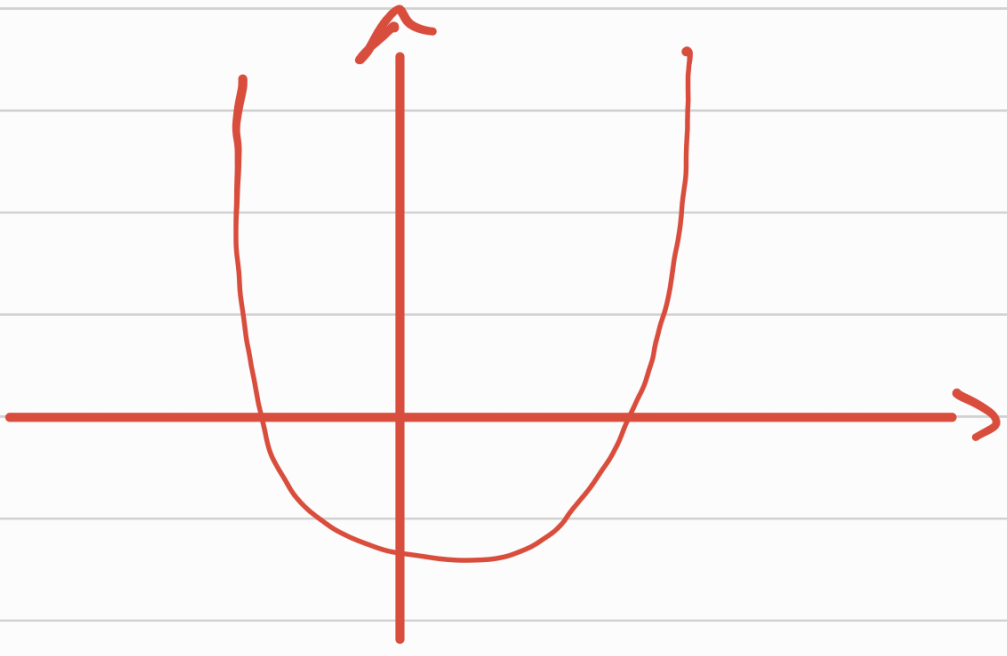
$$\beta = f(\alpha) = f(5) = -40$$

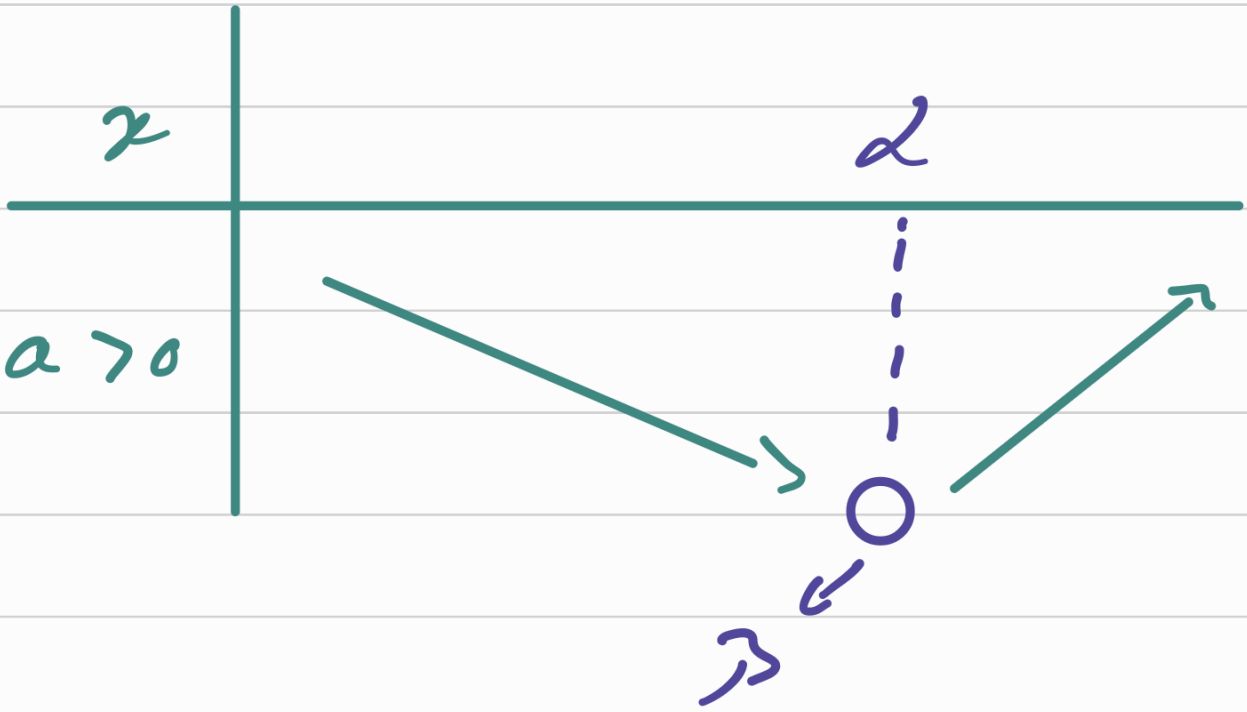
donc $f(x) = 2(x-5)^2 - 40$

a ↙ α ↙ β ↙

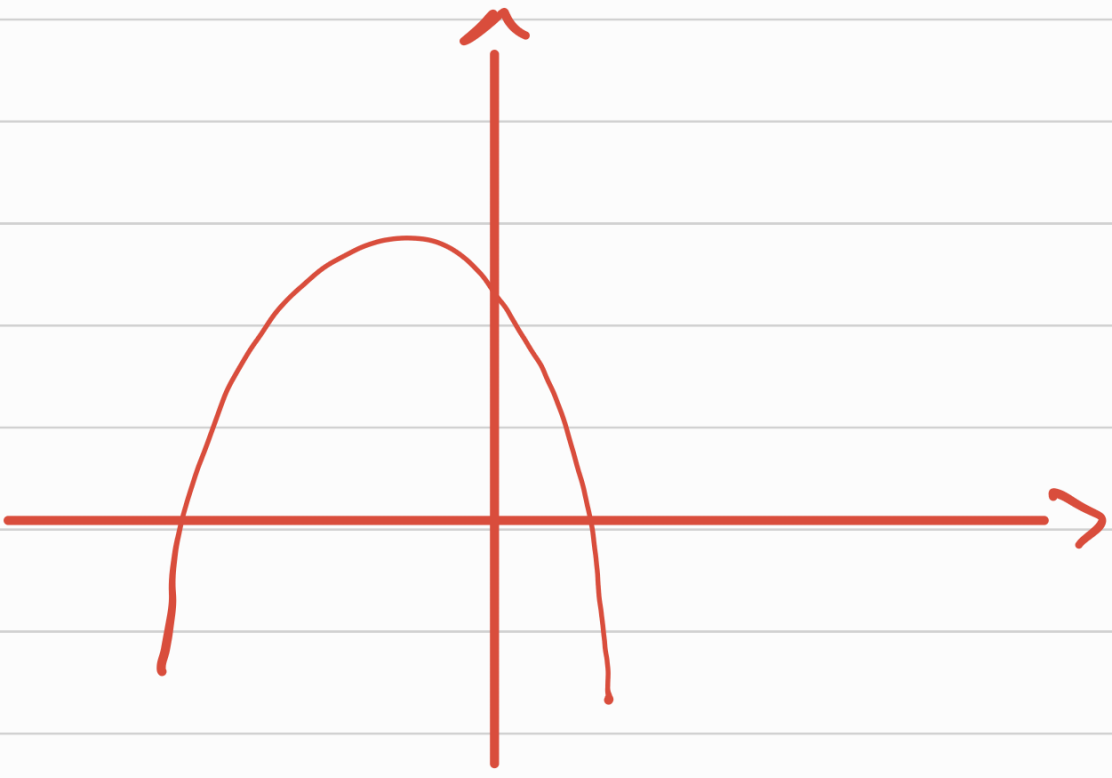
• Lien graphique

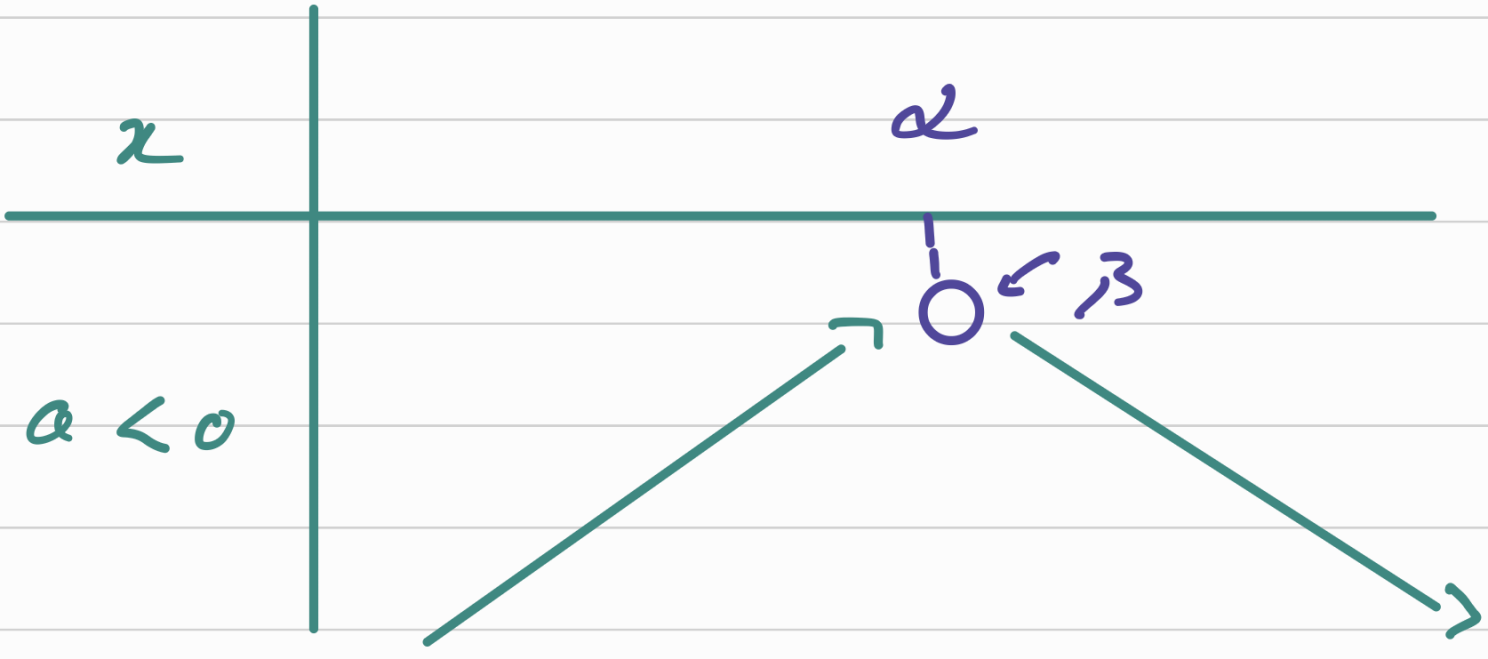
→ Si $a > 0$, alors :





→ $a < 0$, alors :





Donc :

→ si $a > 0$, alors f admet un minimum en $x = \alpha$ et son minimum vaut β.

→ si $a < 0$, alors f admet un maximum en $x = \alpha$ et son maximum vaut β

→ f admet une symétrie axiale d'axe : $x = \alpha$

exercice :

$$f(x) = 2x^2 - 12x + 1$$

$$\alpha = \frac{-b}{2a} = \frac{-(-12)}{2 \times 2}$$

$$= \frac{12}{4}$$

$$= 3$$

$$\beta = f(3) = 2 \times 3^2 - 12 \times 3 + 1$$

$$= -17$$

$$f(x) = 2(x-3)^2 - 17$$

$$f(x) = 4x^2 - 24x + 20$$

$$\alpha = \frac{-b}{2a} = \frac{-(-24)}{2 \times 4}$$

$$= \frac{24}{8}$$

$$= 3$$

$$\beta = f(3) = -16$$

$$f(x) = 4(x-3)^2 - 16$$