exercice 13 :

$$F = \{(x,y,z) : x - 2y + z = 0\}.$$

$$= \{(x,y,z) : (x,y,z).(1,-2,1) = 0\}$$

on pose $\underbrace{(1,-2,1)}^{w}$

$$= \left(\mathrm{Vect}\,\{\overbrace{(1,-2,1)}^{w}\}\right)^{\perp} = \{w\}^{\perp}$$

$$\boxed{A^{\perp} = \{x \in E : x.a = 0,\ \forall a \in A\}}$$

$$\dim\left(\mathrm{Vect}\,\{(1;-2,1)\}\right) = 1$$

$$\hookrightarrow \dim F = \underset{\mathbb{R}^3}{3} - \underset{\mathrm{Vect}}{1} = 2$$

$$A \oplus A^{\perp} = E$$

$\hookrightarrow \dim A^{\perp} + \dim A = \dim E$

$$E_1 \oplus E_2 = E$$

$$\Longleftrightarrow \begin{cases} E_1 \cap E_2 = \{0_E\} \\ \dim E_1 + \dim E_2 = \dim E \end{cases}$$

Soit $u(x; y; z) \in \mathbb{R}^3$

$$\Pi_F(u) = u - \frac{w \cdot u}{\|w\|^2} \times w$$

cte

$$\omega \cdot u = (1,-2,1) \cdot (x,y,z)$$
$$= x - 2y + z$$

$$\|\omega\|^2 = 1^2 + (-2)^2 + 1^2 = 6$$

donc :

$$\Pi_F(u) = (x,y,z) - \frac{x-2y+z}{6}(1,-2,1)$$

$$= (x,y,z) - \left(\frac{x-2y+z}{6} ; \frac{-2x+4y-2z}{6} ; \frac{x-2y+z}{6}\right)$$

$\times 1$ $\quad$ $\times -2$

$$= \left(x - \frac{(x - 2y + z)}{6} ;\ y - \frac{(-2x + 4y - 2z)}{6} ;\ z - \frac{(x - 2y + z)}{6} \right)$$

$$= \left(\frac{5x + 2y - z}{6} ;\ \frac{2x + 2y + 2z}{6} ;\ \frac{-x + 2y + 5z}{6} \right)$$

$$= \left(\frac{5}{6}x + \frac{y}{3} - \frac{z}{6} ;\ \frac{1}{3}x + \frac{1}{3}y + \frac{1}{3}z ;\ -\frac{x}{6} + \frac{y}{3} + \frac{5}{6}z \right).$$

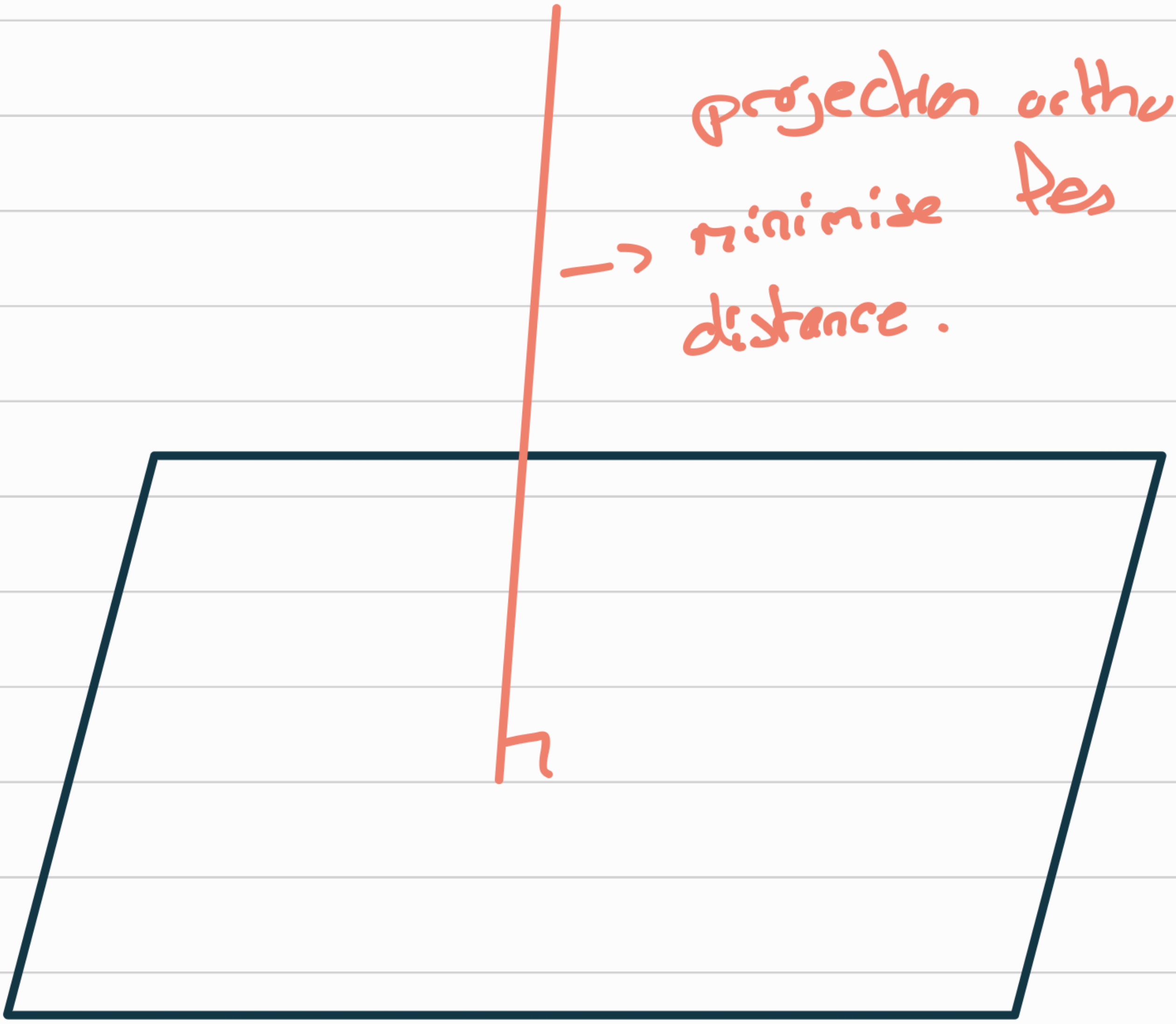
$$\mathrm{Mat}(\pi_F) = \begin{pmatrix} \frac{5}{6} & \frac{1}{3} & -\frac{1}{6} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{6} & \frac{1}{3} & \frac{5}{6} \end{pmatrix}$$

2) $$\mathrm{dist}((1,-2,0);F) = \frac{|\overset{\omega}{(1,-2,1)}.(1,-2,0)|}{\|\underset{\omega}{(1,-2,1)}\|}$$

$$|(1,-2,1).(1,-2,0)| = 5$$

$$\|(1,-2,1)\| = \sqrt{6}$$

$$\mathrm{dist}((1,-2,0);F) = \frac{5}{\sqrt{6}}$$


projection on the plane
minimise distance.

exercice 14 :

$$D = \left\{ x = \frac{y}{3} = \frac{z}{5} \right\}.$$

1) $D = \mathrm{Vect}\{(1,3,5)\}.$

$= \mathrm{Vect}\left\{ \frac{1}{\sqrt{35}}(1,3,5) \right\}$

$= \mathrm{Vect}\{e\}$

Soit $u(x;y;z) \in \mathbb{R}^3$.

$\pi_D(u) = (u.e) \times e$

(x,y,z)

$= \frac{1}{\sqrt{35}}(x+3y+5z)\left(\frac{1}{\sqrt{35}};\right.$

$\left.\frac{3}{\sqrt{35}}; \frac{5}{\sqrt{35}}\right)$

$$= \frac{1}{35}\left(x+3y+5z\,;\,3x+9y+15z\,;\,5x+15y+25z\right).$$

$$M(\pi_D) = \frac{1}{35}\begin{pmatrix} 1 & 3 & 5 \\ 3 & 9 & 15 \\ 5 & 15 & 25 \end{pmatrix}$$

2) $\mathrm{dist}\big((\overset{w}{3;-1,1}), D\big)$

$$= \|\overset{w}{(3;-1;1)} - \overset{\pi_D(w)}{\pi_D(3;-1,1)}\|$$

$$= \|(3,-1,1) - \underbrace{\tfrac{1}{35}(5;15;25)}_{\left(\frac{1}{7};\,\frac{3}{7};\,\frac{5}{7}\right)}\|$$

$$= \left\| \left(\frac{20}{7} ; -\frac{10}{7} ; \frac{2}{7} \right) \right\|$$

$$= \sqrt{\left(\frac{20}{7}\right)^2 + \left(-\frac{10}{7}\right)^2 + \frac{2^2}{7}}$$

$$= \frac{6\sqrt{14}}{7}$$